DETROIT PUBLIC SCHOOLS

Department of Instruction,
Teacher Training and Research

ELEMENTARY PROJECTION, DEVELOPMENTS
AND
SHEET METAL DRAFTING

Published by the Authority of the
Board of Education
City of Detroit
1921

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ORTHOGRAPHIC PROJECTION

Since all working drawings are made upon the principles of orthographic projection it is necessary before proceeding further to make a rather careful study of those principles. Orthographic projection simply means the drawing out on paper of forms showing their exact form and position with relation to co-ordinate planes.

Perspective:—Looking at an object, visual rays extend from points of the object to the eye, all converging at the eye. If a transparent screen is placed between the object and the eye a picture is formed on the screen. This picture, or perspective, never appears as the object actually is. Thus, by perspective, it is impossible to show proper proportions or exact dimensions. Fig. 1.

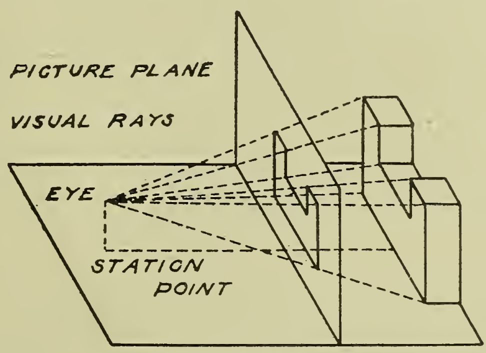

FIG. 1. PERSPECTIVE

Orthographic Projection:—In perspective the point of sight is always at a given or finite distance from the object, while in orthographic projection the point of sight is considered at an infinite distance and the object at a finite distance from the screen, or plane. Hence the visual rays are considered perpendicular to the plane, or screen, and parallel to each

other. Consequently all forms appear in their exact relation to the plane. Fig. 2.

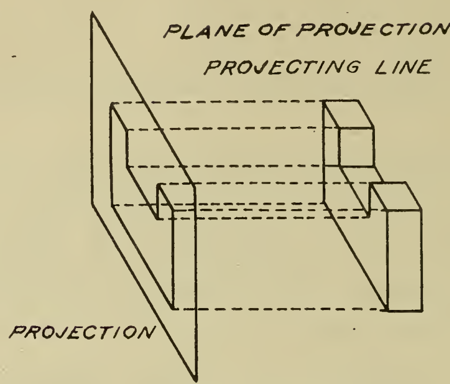

FIG. 2. ORTHOGRAPHIC PROJECTION

Terms Used:—In order to discuss the principles of orthographic projection it will be necessary to define certain terms used in the discussion. Let a point, as **B**, be assumed in space. Fig. 3. A straight line drawn from the point perpendicular to a plane is the **projecting line, B b′.** The plane to which it is drawn is the **plane of projection.** The point where the line meets, or pierces, the plane is the projection of the point assumed, b′. Thus it will be seen that a projecting line is nothing more or less than a visual ray from the point assumed to the point of sight, at an infinite distance and the projection is the piercing point of the visual ray.

Planes of Projection:—A **horizontal** plane of projection and a **vertical plane** of projection are assumed in orthographic projection. These are mutually perpendicular. Projecting lines are drawn perpendicular to these planes of projection. Hence projecting lines are either horizontal or vertical. The projection of an object on the vertical plane of projection is its **vertical projection, b′, h′, k′, c′,** Fig. 3. The projection on the horizontal plane is its **horizontal projection a, b, c, d.** Projections are named from the planes on which they are made.

A plane of projection perpendicular to both the horizontal and vertical planes is often required. This plane is called the **profile plane of projection**. A projection made upon this plane is called a **profile projection**, a″, f″, h″, b″, Fig. 3.

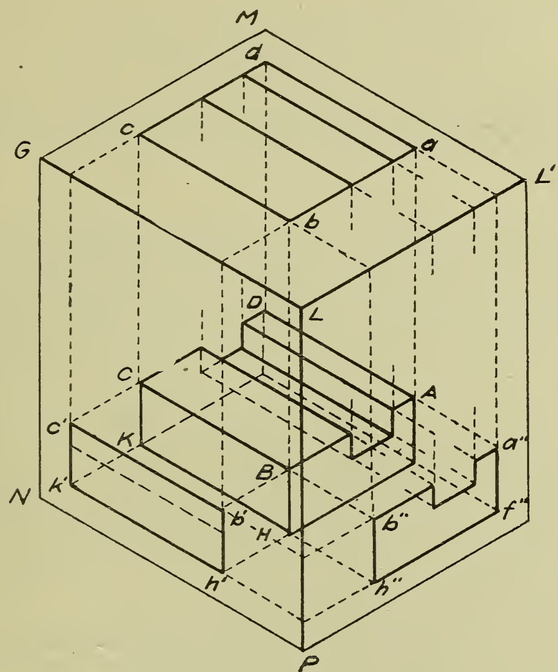

FIG. 3

An auxiliary plane is frequently used to give a view of the exact shape and size of a surface that cannot be shown on the horizontal, vertical, or profile planes.

Views:—View in connection with working drawings means a projection. The horizontal projection of an object is often called the **Top view** or **Plan**; the vertical projection is called a **Front view** or **Elevation**; and the profile projection is called an **End view**, or an **End Elevation**.

Projection of Lines, Surfaces, and Solids:—Since a line is determined by its points, its projection is determined by the projection of these points on the planes of projection. When the extremities of a straight line have been projected onto a plane of projection, a straight line joining these projections is the projection of the line. Observe in Fig. 3 that the projec-

tions of the points B and C at b and c give the extremities of the horizontal projection of the line BC.

Since a surface is bounded or determined by lines, the projections of these bounding lines determine the projection of the surface.

A solid is determined by bounding surfaces, and its projections by the projections of the bounding surfaces.

Therefore to find the projection of a solid, first find the projection of the points determining the lines which bound the determining surfaces of the solid, then connect these with the proper lines.

Revolving the Planes:—A drawing is made in one plane, the plane of the paper. It is therefore necessary to revolve that portion of the H plane behind the vertical about its intersection with the V plane until it becomes above and in the same plane with the V plane, Fig. 4. The intersection of the

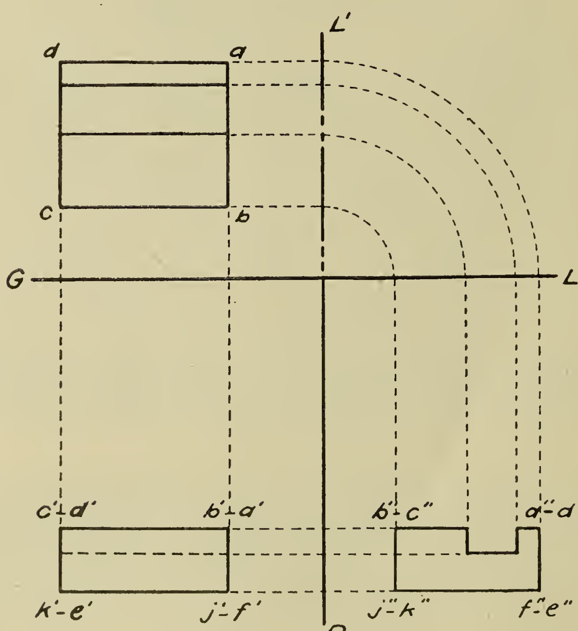

FIG. 4

H plane and the V plane is called the **ground line (G. L.)** On the drawing the ground line is represented by a horizontal line. All that portion of the paper above or behind the G. L. represents the H plane while all that portion below the G. L. represents the V plane.

In the same manner, the Profile plane is revolved about its intersection with the V plane until it is in the same plane with the V plane. The intersection of the Profile plane and the vertical plane is called the profile line (P. L.) On the drawing the P. L. is represented by a vertical line and therefore perpendicular to G. L. Since the P plane is placed either to the right or the left of the object according to the view or projection required, the P plane may be revolved to the right or left, as the case may be.

Fig. 3 illustrates a piece, A B C D E F J K, located in the third angle below H and behind V with the Profile plane at the right. The horizontal projection of the grooved block is shown at a b c d, the vertical projection at c′ b′ j′ k′, and the profile projection at a″ f″ j″ b″. When the H plane, M L′ L G has been revolved about G. L. and the V plane, L L′ N P has been revolved to the right about P. L., the H, V, and P planes all lie in the same plane. This may be taken as the plane of the paper. Fig. 4 illustrates the position of the projections when they have been brought into the same plane. Observe that the H and V projections of any given point on the piece lie in the same vertical line, and that the V and the P projections lie in the same horizontal line.

Third Angle:—The horizontal and vertical planes (considered infinite in extent) are perpendicular to each other and form four right dihedral angles, Fig. 5. Let the horizontal plane be denoted by H and the vertical plane by V. An object lies in the

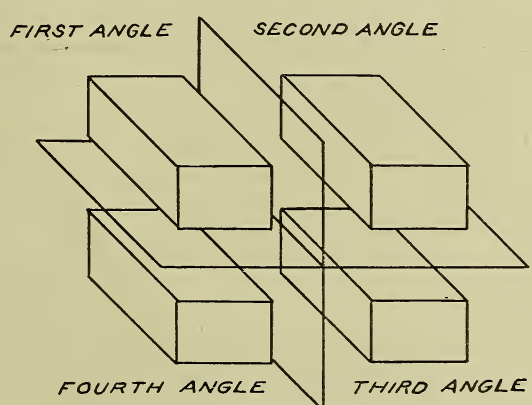

FIG. 5

First angle when it is above H and in front of V.

Second angle when it is above H and behind V.

Third angle when it is below H and behind V.

Fourth angle when it is below H and in front of V.

It is almost the universal practice in this country to use the third angle since it affords a more convenient arrangement of the views or projections for working drawings and is more modern. The first angle is still used to some extent in this country and almost universally in European countries, although even there the use of the third angle is slowly becoming more common.

The third angle unless otherwise noted is used in this discussion and in the problems for practice.

Relative Position of Views—Any point in the third angle has its H projection above the ground line, its V projection below the Ground Line in the same vertical line with the H projection. It may further be stated that the profile projection and the vertical projection are in the same horizontal line.

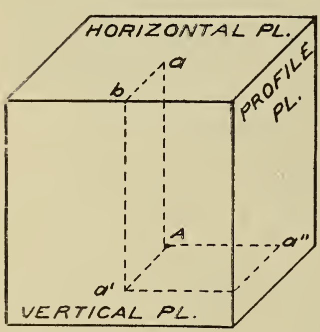

FIG. 6

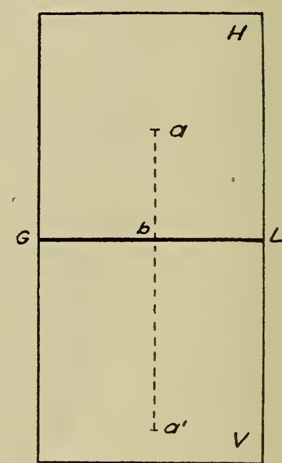

FIG. 7

In Fig. 6, if A is a point in space then Aa and Aa′ are projecting lines which determine the H projection and the V projection, respectively. Now, if a plane is passed through the point A and also through the projecting lines Aa and Aa′, it will intersect H on the line ab and V on the line a′b, both of which are perpendicular to the G. L. When H and V have been revolved into the same plane ab and a′b will lie in the same straight line perpendicular to the Ground Line. Fig. 7.

In drawing, when one projection has been determined, the other may be determined by drawing a straight line perpendicular to the G. L. through the projection first found and locating the other projection on this line the proper distance from the G. L. The line so drawn is a Line of Projection and should not be confused with a Projecting Line. A line of projection exists only on the drawing; a projecting line exists only in imagination. A line of projection might be said to be a projection of a projecting line. Observe in Fig. 3, that B b is a projecting line, and that, in Fig. 4, b b′ is a line of projection.

A draftsman must first be able to conceive each view or projection on its corresponding plane of projection and then determine the positions of the different views with respect to each other and to the Ground Line when the planes of projection have been brought into the same plane. This requires considerable practice and should become so natural that one should not be conscious of the first step at all.

PRINCIPLES OF PROJECTION

A Point in Space

(1) Every point has two views, one on H, the other on V, by which its position in space is determined. Fig. 6.

(2) A point and its two views lie in the same plane perpendicular to both H and V—i.e., a plane passed through the two projecting lines. Fig. 6.

Since the projecting line, Aa in Fig. 6, is perpendicular to H a plane passed through Aa is perpendicular to H. Similarly Aa′ is perpendicular to V. Therefore a plane passed through the point A and its two projecting lines is perpendicular to both H and V. This plane is called a Projecting plane.

(3) The Top view of a point is as far behind the Ground Line as the point itself is behind V. The Front view of a point is as far below the Ground Line as the point itself is below H. Fig. 4.

(4) The two views of a point always lie in the same straight line, perpendicular to the Ground Line. Fig. 7.

The intersections, ab, with H, of the plane passed through Aa is perpendicular to V and to the Ground Line at b and the

intersection, a'b with V of the plane passed through Aa is perpendicular to H and the Ground Line at b. Therefore when the horizontal plane (H) has been revolved until it is in the same plane with V, ab and a'b will lie in the same straight line perpendicular to the Ground Line. Both ab and a'b are perpendicular to the Ground Line and have the point b in common.

A Line in Space

(5) A line perpendicular to either plane of projection has for its view on that plane simply a point, Fig. 8.

(6) A line perpendicular to either plane of projection has for its view on the other plane a straight line perpendicular to the Ground Line and equal in length to the line of which it is the projection, Fig. 8.

It is evident that the intersection of the plane of projection and the projecting plane of the line is perpendicular to the Ground Line.

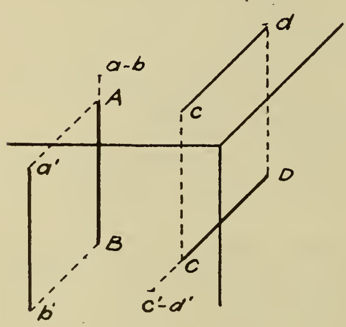

FIG. 8

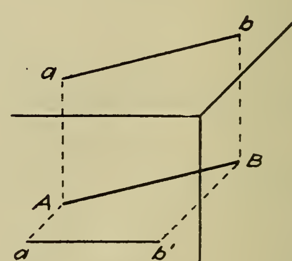

FIG. 9

(7) When a line is parallel to either plane of projection its view on that plane represents the true length of the line; and the angle which this view makes with the Ground Line is equal to the angle which the line in space makes with the plane to which it is not parallel, Fig. 9.

(8) A line parallel to either plane of projection has for its view on the other plane a line parallel to the Ground Line, Fig. 9.

(9) When a line is not parallel to a plane of projection, its view on that plane is always shorter than the true length of the line, Fig. 9.

(10) A line parallel to both H and V has for its two views, lines parallel to the Ground Line, both of which are equal in length to the line itself.

(11) If a line is parallel to neither plane of projection, both views are shorter than the line itself. The angles which the line makes with the planes of projection are not represented in their true size by the angles which the views make with the Ground Line, Fig. 10.

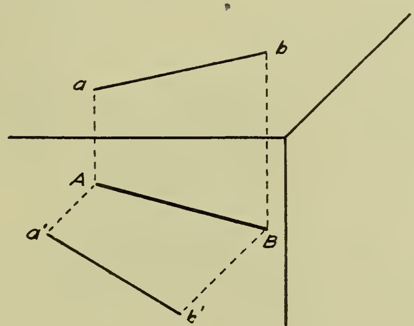

FIG. 10

In general, the projection of a line parallel to a plane of projection is equal to the line itself. And, the projection of a line at an angle with a plane of projection is shorter than the line itself. A projection of a line is either equal to or shorter than the line itself. Since surfaces are bounded by lines, this principle is also true of surfaces.

In order to find the true size and shape of either a line or surface not parallel to the H, V, or P plane of projection, it is necessary to either revolve the line or surface until it is parallel with one of these planes; or assume a plane parallel to the surface or line. A plane so assumed is an auxiliary plane of projection.

AUXILIARY VIEW

It is often necessary to make a view of an object that is in such a position that a suitable view is not shown on the horizontal, vertical, nor profile planes. As in the case of the truncated square prism, Fig. 11, the true size and shape of the section cut by the plane is not shown either on the horizontal,

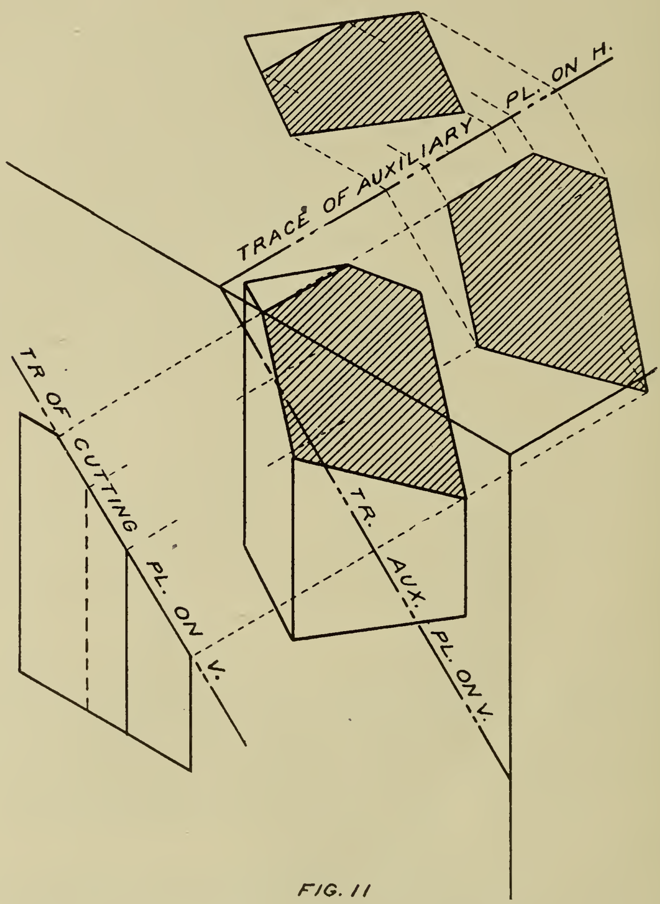

FIG. 11

vertical, or profile plane. A plane, parallel to the cutting plane, is used and the projection is made on this plane. A plane so used, or assumed, is called an **auxiliary plane of projection.** The projection made on this plane, of the section made by the **cutting plane,** will be the true size and shape of the surface, for the surface is parallel to the auxiliary plane.

It will be observed in Fig. 11, that the Trace of the Auxiliary Plane on V is parallel to the Trace of the Cutting Plane on V, for the auxiliary plane is assumed parallel to the cutting plane. Projecting lines drawn from the object to the auxiliary plane of projection are perpendicular to this plane. The traces of a projecting plane, passed through one of these projecting lines perpendicular to the V plane, will form lines of

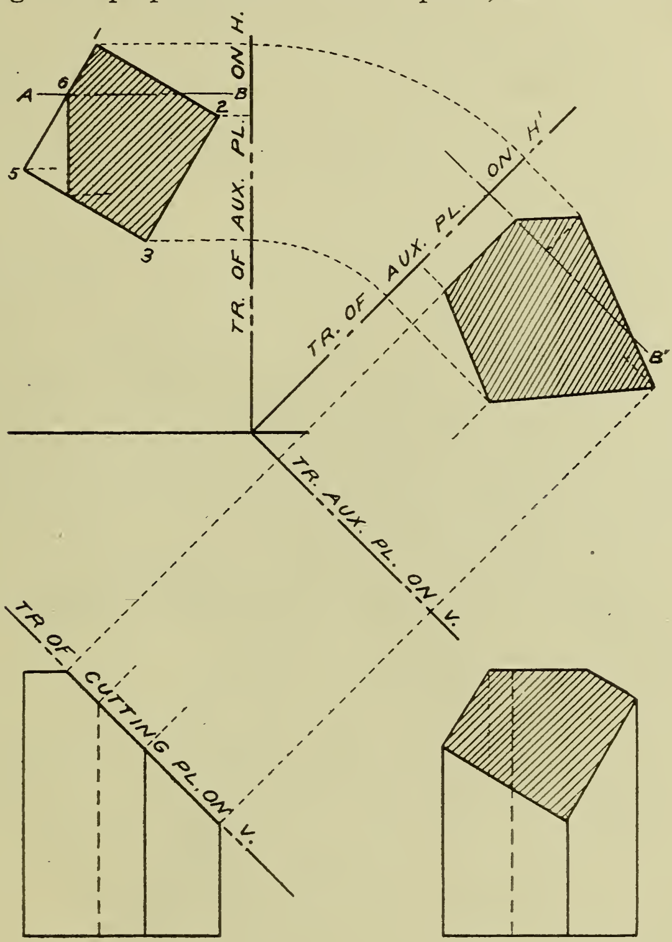

FIG. 12

projection on the auxiliary plane and on the V plane. When the auxiliary plane has been brought into the same plane with H and V, the corresponding lines of projection on V and on the auxiliary plane will lie in the same straight line perpendicular to the auxiliary trace on V and also to the trace of the cutting plane on V, as shown in Fig. 12.

First Method:—Fig. 12. In the drawing, make the Trace of the auxiliary plane on V parallel to the Trace of the cutting plane and also locate the auxiliary trace on H before the planes have been revolved and also when the auxiliary plane has been revolved into the same plane with V. The trace on H will now be perpendicular to the auxiliary trace on V, and is indicated as the trace of auxiliary plane on H′, Fig. 12. Draw the lines of projection to the auxiliary trace on H and with the intersection of this trace with ground line as a center swing the lines of projection to the trace of auxiliary plane on H′. Continue the lines of projection parallel to the auxiliary trace on V. Draw the lines of projection from the V projection perpendicular to the auxiliary trace on V. The auxiliary view is determined by the intersection of the corresponding lines of projection.

Second Method:—Fig. 12. Another method, which is often more convenient, is employed. In this case an axis is drawn through the section made by the cutting plane, the H projection of which is represented by the line AB, Fig. 12. Locate this axis on the auxiliary plane by the line A′B′ parallel to and at a convenient distance from the trace of the cutting plane. Draw the lines of projection from the V projection perpendicular to A′B′. With the dividers, measure the distances on the corresponding lines of projection from the line A′B′ on the auxiliary plane. Connect the points thus found and determine the auxiliary view.

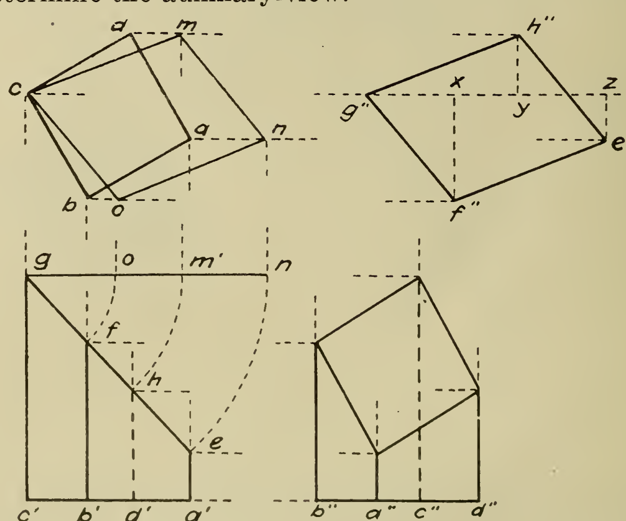

FIG. 13

Third Method:—Fig. 13. Surface revolved into position parallel to H. This method is very convenient and may be

used to great advantage. If the surface bounded by HE, EF, FG, and GH be revolved about G until it is parallel to the H plane then the projections will assume the form shown in Fig. 13. With g as center, revolve ge to the horizontal position, as gn′; the points f and h will fall at o′ and m′. Lines of projection drawn from the points m′, n′, and o′, to the corresponding lines drawn from d, a, and b give the horizontal projection of F, H, and E. Then the parallelogram bounded by the lines cm, mn, no, oc is the horizontal projection of the surface after it has been revolved into a position parallel to H. It is more convenient however to place this view to one side, as follows:

Simply project horizontal lines from the points a, b, c, and d. Locate g″ at a convenient position on cz, set off g″x equal to gf, xy equal to fh, and yz equal to he. At x draw xf″ perpendicular to g″z; at y, the perpendicular yh″ and at z, the perpendicular ze″. The intersection of the line bf″ and xf″ is the location of the point f″; h″ and e″ are found in the same manner.

True Length of Line:—In Fig. 14, let AB be a line parallel to neither H nor V. Then ab and a′b′ are its projections on the horizontal and vertical planes respectively. If the point A remain fixed and the line be revolved until it is parallel to V the point B will move to M, in an arc, the plane of which is parallel to H. The projection of this arc on H is bm and on V is the line b′m′. The horizontal projection of AB in its new position is am parallel to G. L. for the line AB is now parallel to V. The V projection of AB in its new position is a′m′, and is the True Length of AB, for it has been made parallel to V.

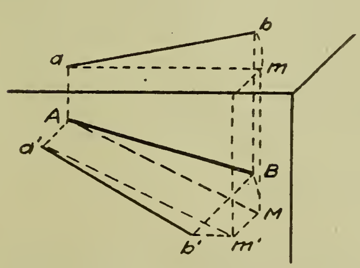

FIG. 14

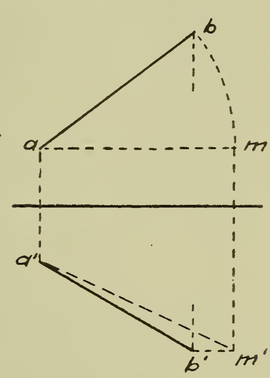

FIG. 15

On the drawing, Fig. 15, using a as center and radius ab, draw the arc bm and the straight line, am, parallel to G. L. Draw b′m′ parallel to G. L. on V. Draw the vertical line mm′

intersecting b′m′ at m′. Connect a′ and m′. Then a′m′ is the True Length of the line. Since the extremity of the line moves in a plane parallel to the plane of projection, any point in the line will move in a plane parallel to this plane and to the plane of projection.

A method used to great advantage in practical work is as follows: In Fig. 16, Ab is the H projection of AB. Bb, the projecting line of B, is perpendicular to the H plane and therefore to Ab. Then AbB forms a right-angled triangle of which AB is the hypotenuse, Bb the altitude, and Ab one leg. When the H plane has been revolved into the same plane, Fig. 17, ab

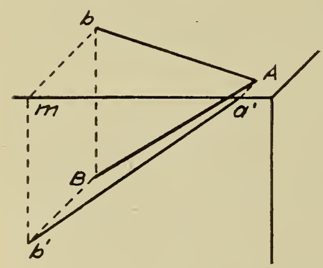

FIG. 16

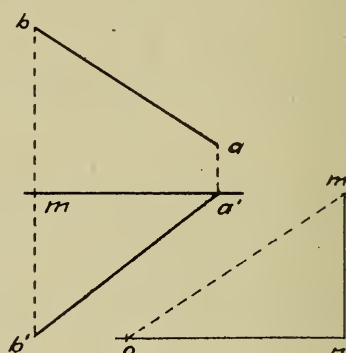

FIG. 17

is the H projection, a′b′ is the V projection, and mb′ is the altitude of the line AB and of the V projection. In the right triangle m′on, Fig. 17, make m′n equal b′m and no equal ab, then m′o is the true length of the line AB. Since the triangles m′on, Fig. 17, and AbB, Fig. 16, are equal by construction, the hypotenuse of one is equal to the hypotenuse of the other.

DEVELOPMENTS

The ability to develop, or lay out, the surface of solids or forms is an important part of a draftsman's equipment. The laying out of such work is a large part of the sheet-metal worker's business. Many of the so-called practical methods or short-cuts employed by the trade may easily be explained by the principles of orthographic projection. A thorough understanding of these principles will make the practical applications comparatively simple.

To lay out the surface for a truncated prism:—In Fig. 18, since all the edges of the prism are perpendicular to the base, straight lines representing the edges are drawn at right angles to a straight line representing the distance around the prism at the base. To find the points at which to draw these perpendiculars, set off a″b″ equal to ab; b″c″ equal to bc; c″d″ equal to cd; and d″a″ equal to da. On these perpendiculars

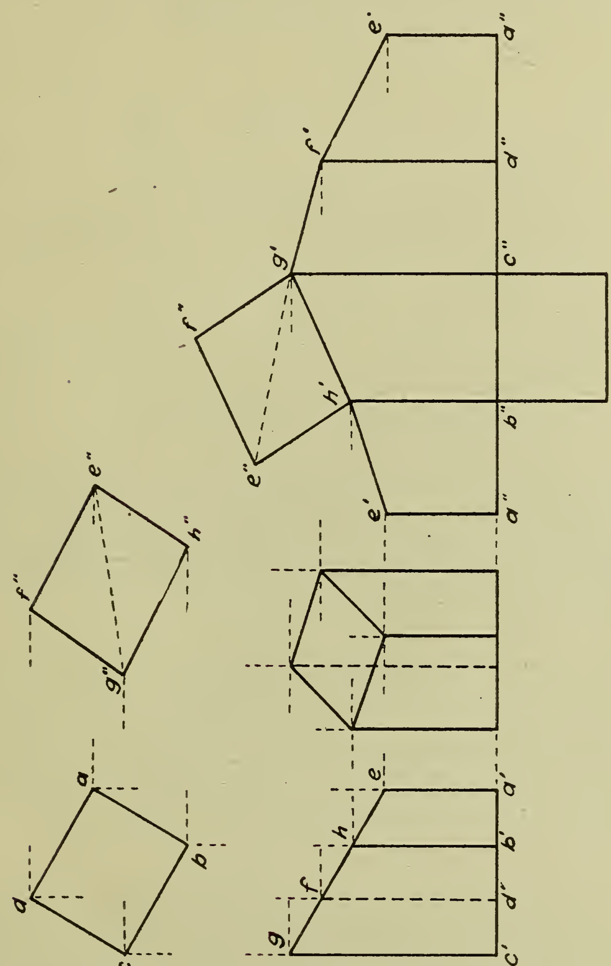

FIG. 18

set off the heights of each edge as a″e′ equal to a′e; b″h′ equal to b′h; etc. Joining the points e′h′, h′g′, etc., the sides are laid out. The bottom and top may be joined in the proper

position. Care must be taken to place these in such a position that when folded into place the edges will meet their respective sides. Those surfaces must be of the true size and shape.

To lay out the surface of a pyramid:—It will be observed that the lateral surfaces of a pyramid are triangular. If the edges bounding these surfaces are determined, the triangles forming the surfaces may be constructed. From an exam-

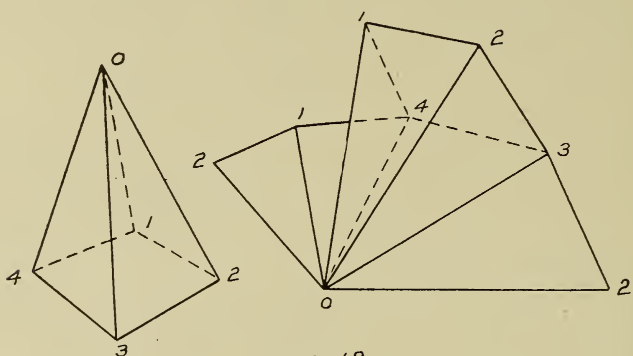

FIG. 19

ination of Fig. 19, it will be observed that the triangles are adjacent with a common point at the vertex, one side in common, and that the bases are not in a straight line. Observe that the length of the sides of the triangles is the true length of the edges of the pyramid in each case.

In Fig. 20, are shown the projections of a truncated square pyramid. In order to find the development of its surface it is necessary to determine the true length of the edges. The true length of **0-2** is found to be **$0'$-2_1**. 2_1 is found at the intersection of a horizontal line drawn through $2'$ and **$0'$-2_1**. **6_1** is found at the intersection of a horizontal line drawn through $6'$. (See true length of line, Fig. 15).

Since all the edges in this right pyramid are equal, the arc **2_1-2_1** is drawn with **$0'$** as center and **$0'$-2_1** as radius. With the chord **1-2** equal to one side of the base, set off the distances **2_1-1_1, 1_1-4_1, 4_1-3_1** and **3_1-2_1**. Set off the distance **$0'$-6_1** on the line **$0'$-2_1** in the development equal to **$0'$-6_1; $0'$-1_1, $0'$-8_1** equal to **$0'$-7_1**. Other points are found in a similar manner. Complete the lay-out by joining the points and adding the truncated surface as well as the base.

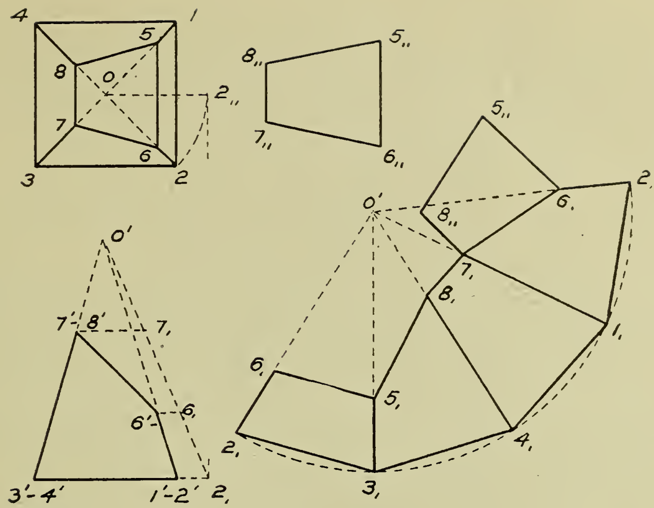

FIG. 20

CURVED SURFACES

A surface may be generated, or produced by moving a line. If the moving line be made to meet certain conditions or follow certain lines, definite surfaces may be generated.

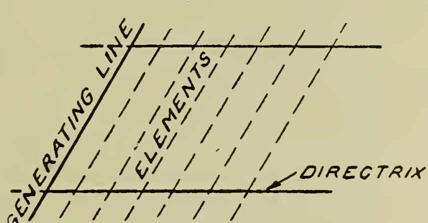

FIG. 21

A straight line moving so as to touch two parallel lines would produce a plane surface. The moving line is the **generating** line. The line giving the direction to the moving line is the **directrix**, the various successive positions of the generating line are the **elements**, Fig. 21.

Cylinder

Every cylinder may be generated by moving a straight, or right line, so as to touch a closed curve and have all its positions parallel. If a straight line be moved so that it touches a circle and its various positions are parallel to each other and perpendicular to the plane of the circle, a right circular cylinder will be generated. The moving line is the generating line; the various positions of the moving line form elements on the curved surface of the cylinder.

The consideration of these elements on the curved surface of the cylinder is essential to the development of the surface, especially that of a truncated cylinder or of intersecting surfaces. In the drawing, to locate any point on the curved surface, it is necessary to locate the element passing through this point. The element being located on any one of the different projections, it is easy to locate this point on the given element in any one or all of the views.

If any point as A, Fig. 22, be located on the surface of a cylinder the vertical projection of which is a′ a straight line drawn through a′ parallel to the axis, will represent its element. The point a will not only be the horizontal projection of the point A but also of its element. The profile projection of the element may be made and the location of a″ may be projected from a′.

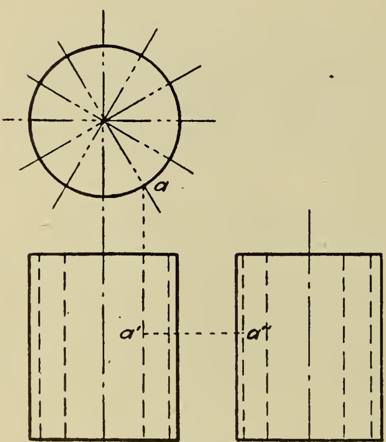

FIG. 22

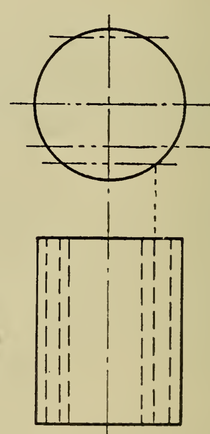

FIG. 23

To determine elements on the surface of a cylinder:—A plane passed through a cylinder so as to contain the axis will intersect the surface in two straight lines parallel to the axis. Since these two lines are parallel to the axis and therefore to each other, they are right line elements on the curved surface, Fig. 22. Again a plane may be passed through a cylinder

parallel to the axis and its intersection with the curved surface will be straight lines parallel to the surface and to each other. These lines, then, are **right line elements** of the curved surface, Fig. 23.

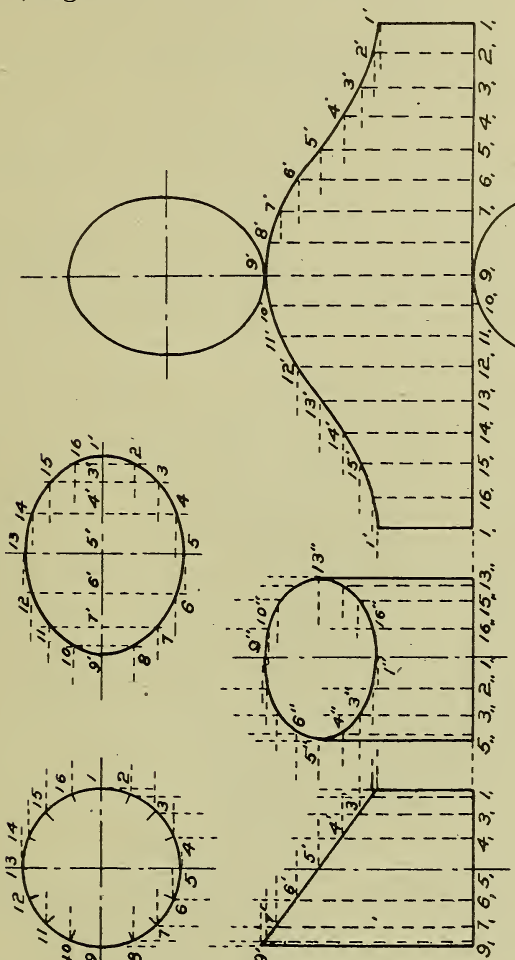

FIG. 24

Right line elements, then, may be found on the surface of a cylinder in two ways: (1) By passing a plane through the cylinder parallel to the axis; or, (2), By passing a plane through the axis in any direction so that the axis is contained in the plane. Fig. 24 readily illustrates the method of laying out the surface of a cylinder. How long must the line 1_1-1_1 be? Why must the distances 1_1-2_1, 2_1-3_1, etc., on the development, be taken from the circumference in the **H** projection? Knowing the diameter, what is the formula for figuring the circumference?

Cone

Every cone may be generated by moving a right, or straight, line so as to touch continually a closed curve and pass through a fixed point not in the plane of the curve.

The moving line is the **generating** line; the fixed point the vertex of the cone; and the different positions of the generating line are the right-line **elements.**

These right-line elements may be found on the surface of a cone by passing a plane either through its vertex in any direction so as to contain the axis of the cone, or through the vertex and the base of the cone in any direction.

Elements thus found may be shown on the drawing by dividing the circumference of the base into a number of parts, preferably equal parts, and drawing lines from these points through the vertex as in Fig. 25. These lines represent the

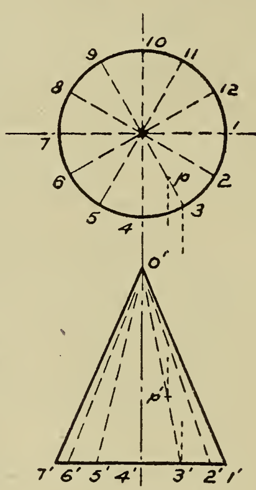

FIG. 25

elements in the **H** projection. Project the points **2, 3, 4, 5,** etc., to the base in the vertical projection as **2′, 3′, 5′,** etc. Connect the points thus found with **0′**, the apex.

If **a′** in Fig. 26, is the **V** projection of a point on one of the elements as **0′-3′**, then its **H** projection may be found by projecting its position from **a′** to its position on **0-3** at **a.** Again if any point as **P** has its **V** projection at **p′**, Fig. 25, a straight line representing an element is drawn from the apex through **p′** to the base. A projection of this element is then made on the horizontal projection and the point **p** found by projecting from **p′**.

To develop a right circular cone:—It is evident that in a right circular cone all the elements are of equal length. The line **$0'$-1_1** is the true length of one of these elements, Fig. 26. Why? With a radius **$0'$-1_1**, draw an arc setting off distances **1_1-2_1**, **2_1-3_1** equal to the divisions on the circumference in the H projection. Why? These points may be connected with the center of the arc representing the apex. The length of the arc **1_1-1_1** must equal the circumference of the base.

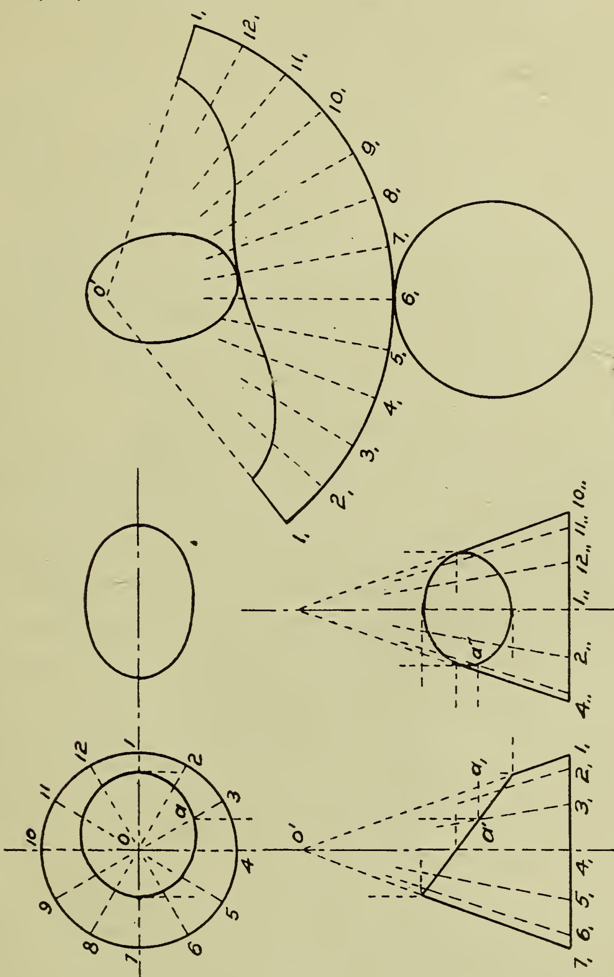

FIG. 26

Since all the elements are equal, the true length of the element **0-3** coincides with **$0'$-1_1**, and the true length of the distance **0-a** will be shown at **$0'$-a_1**. Why? See the pyramid, Fig. 20. The true distance from the apex to any point on any of the other elements may be found in the same way.

INTERSECTIONS

At this stage, the notion should be firmly fixed that a surface is made up of elements. When two surfaces intersect, a point on the line of intersection is the point of intersection of an element of one surface with that of the second surface. These elements are determined by the passing of a plane through the solids in such a way as to determine elements on the surfaces of each. It is evident that an element 'cut out' by a plane on one surface will intersect an element 'cut out' by the same plane on the other surface; since, in a plane, all straight lines not parallel to each other intersect. The point of intersection of the two elements is a point common to both surfaces and is a point on the line of intersection.

The method of finding the intersection of surfaces must be familiar to the well-equipped draftsman, as intersections are constantly occurring both in working drawings, and in sheet metal work. In sheet metal work enough points must be found to lay out the development accurately.

Intersection of Cylinders

If a plane be passed through the two intersecting cylinders, Fig. 27, parallel to the axis of each, the element **AB** will be determined on the large cylinder **G**, and also the elements **CD** and **EF** on the smaller, **H**. It will be noted that **CD** and **EF** intersect **AB** at **D** and **F**, points on the line of intersection. On the working drawing the plane **MM** 'cuts out' the element represented by $c'' b''$ on the smaller cylinder in the profile view and $a' b'$ on the larger cylinder in the front view.

The element **CD** appears in the front view at $c' d'$, obtained by projecting from the top view. The intersection of $c' d'$ and $a' b'$ at d' is a point on the line of intersection of the two cylinders; the point f' is obtained similarly. Other points are found in like manner.

It is more convenient to divide the circumference of the small cylinder into equal parts and so pass the planes as to determine elements equi-distant apart on the smaller cylinder. Time may be saved by drawing a semi-circumference on the **H** and the profile projections.

In laying out the development of the surface of the small cylinder, it is necessary to set off along the straight line, 5_1-5_1 distances equal to the length of the arc **1-2**. As many dis-

tances must be set off as the whole circumference of the cylinder has been divided into, making sure to make the 'lay-out' check with the calculated circumference. At these points erect perpendiculars representing the elements. On these elements set off distances equal to the length of the elements, as obtained from either the horizontal or profile projection of the cylinder H.

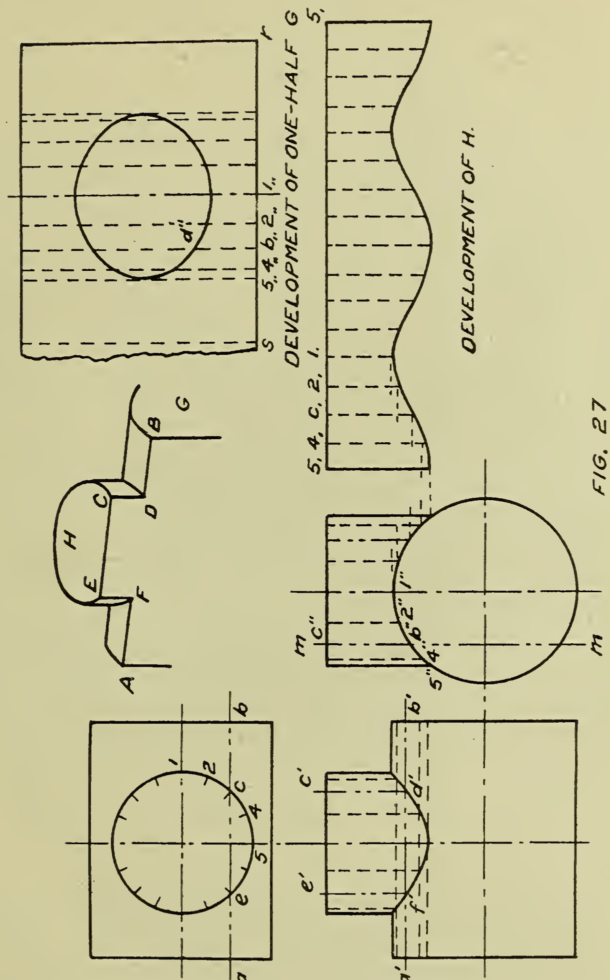

FIG. 27

In laying out the larger cylinder, indicating the center of the opening, at 1_{11}, erect a perpendicular to the straight line rs representing the circumference of the cylinder. From 1_{11} set

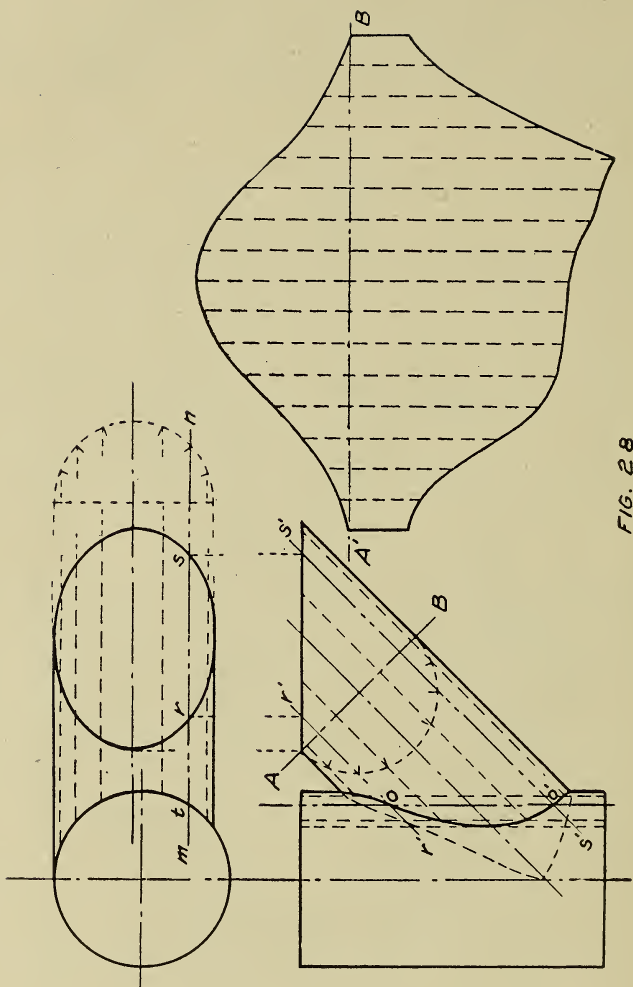

FIG. 28

off on each side along the distance 1_{11}-2_{11} equal to the arc $1''$-$2''$, the distance 2_{11}-d_{11} equal to the arc $2''$-d'', continuing thus set off the remaining distances equal to the corresponding arcs, being sure that the whole distance from r is equal to one-half the figured circumference of the cylinder. From the vertical projection is obtained the distances on the elements as is indicated.

Cylinders at an angle:—The intersection of two cylinders at an angle is found in a similar manner to that of those at a right angle. In Fig. 28, let mn, in the horizontal projection, indicate the trace of the plane MN passed through the two cylinders parallel to the axis of each. The elements 'cut out' on the oblique cylinder are shown in the H projection by rt and st; in the V projection by r′r′ and s′s′. The element 'cut out' on the vertical cylinder is shown in the V projection by t′t′. The points at which r′r′ and s′s′ intersect t′t′ as o and o′ are points on the intersection of the two cylinders. Other points on the intersection may be found in a similar manner.

By means of the auxiliary semi-circle in the H projection the planes for 'cutting out' elements determine these at equal distances on the surface of the oblique cylinder. A similar semi-circle in the V projection serves to locate the position of the corresponding elements in that projection.

In laying out the oblique cylinder, draw AB perpendicular to the axis. Let A′B′, in the development, represent this line. Along A′B′ set off equal distances between the elements as determined from the auxiliary semi-circle; draw the elements perpendicular to A′B′; and set off from A′B′, along the elements, distances equal to the corresponding distance from AB in the vertical projection.

PROBLEMS

Show the Horizontal and Vertical Projections

Prob. 1. (a) A 2¼″ square card is parallel to H with one edge parallel to V. Front edge is ½″ behind V and ⅜″ below H.

(b) The same card, still having the front edge parallel to V, is turned downward on the right 30°.

Prob. 2. (a) A 1¾″ by 2¼″ rectangular card is parallel to V with the longer edge parallel to H. The card is ¾″ behind V; its upper edge ½″ below H.

(b) The same card, the lower edge remaining parallel to H, is turned backward on the right 30°.

Prob. 3. (a) An equilateral triangular card is parallel to V with its lower edge parallel to H. Edges 2″. It is ⅜″ behind V and the lower edge 2¼″ below H.

(b) The same card, except that the lower edge is turned downward on the right 15° to H.

Prob. 4. (a) A 1½″ by 2″ rectangular card is parallel to V. The long edges make an angle of 30° with H. The card is ½″ behind V and its nearest corner ½″ below H.

(b) The same card, except that it is turned back on the right 30° to V.

Prob. 5. (a) A 2″ square card is parallel to H. The edges make an angle of 45° with V. It is ½″ below H and its nearest corner ¼″ behind V.

(b) The same card, except that it is turned down on the right 30° to H.

Prob. 6. (a) An hexagonal plinth, ⅜″ thick, is parallel to V with the upper edge parallel to H. Its sides are 1¼″. It is ⅜″ behind V and its upper edge ½″ below H.

(b) The same plinth turned backward 45° on the right.

Prob. 7. (a) A 1¾″ square plinth, ½″ thick, is parallel to H with the right hand edge 60° to V. The upper surface of the plinth is ½″ below H; the nearest corner ⅜″ behind V.

(b) The same plinth in the same position, except that it is turned downward on the left 30° to H.

Prob. 8. (a) An octagonal plinth, ½″ thick, is parallel to H, the extreme left-hand edge is 75° to V. This plinth is 3″ across from side to side, is 1″ below H, and the nearest corner of the enclosing square is ½″ behind V.

(b) The same pinth, except that it is turned downward on the right 30°.

Prob. 9. (a) A circular plinth, ½″ thick and 2¾″ in diameter; the center of which is 1¾″ behind V and ⅜″ below

H, is parallel to H.

(b) The same plinth turned down on the left 30° to H.

In addition to the H and V projections show the Profile projections in the next six (6) problems. **See page 7.**

Prob. 10. A triangular prism, 3½″ high and sides 2″, 3″, and 3½″ respectively, has the broadest side 75° to the right, the nearest corner ½″ behind V. The base is 4″ below and parallel to H.

Prob. 11. An octagonal prism with 1¼″ sides and 3½″ high rests on an octagonal base parallel to H. One of the sides is 30° on the right with V. The nearest corner is ¾″ behind V and the lower base is 4″ below H.

Prob. 12. A pentagonal pyramid, 3¼″ high and the base inscribed in a 3″ circle, has its base parallel to H and the nearer side of base parallel to V. The base is 4″ below H, the center of which is 2″ behind V.

Prob. 13. A triangular prism, 3″ long, with sides 4″, 2½″, and 4″, rests on the 2½″ by 3″ side with the bases parallel to P. The upper edge is parallel to and 2½″ behind V; and parallel to and ½″ below H.

Prob. 14. An equiangular triangular pyramid with 1¾″ sides and 3¼″ high, has its base parallel to V and the upper edge of its base 15° to H. Base is 4″ behind V and upper corner is 1¼″ below H.

Prob. 15. An hexagonal pyramid 3½″ high with 1½″ sides at the base, has its base parallel to V and one side of base parallel to H. The base is ¾″ behind V and nearest side ¾″ below H.

Revolution of Objects

Prob. 16. (a) An equilateral triangular plinth, 2′-0″ sides and 1′-6″ thick, rests on a 2′ x 1′-6″ rectangular base parallel to and 3′-0′ below H. The nearer triangular face is parallel to and 1′-0″ back of V. Scale, 1½″=1′-0″.

(b) The same plinth in the same position, except that it is turned up on the left 45° to H.

(c) The same plinth in the same position, except that it is turned back on the right 30° to V.

Prob. 17. (a) A square pyramid, 1′-6″ square at the base and 2′-6″ in altitude, rests on its square base parallel to and 3′-0″ below H so that the sides of the base are at 45° with V, the nearest corner being 6″ back of V. Scale, 1½″=1′-0″.

(b) The same pyramid in the same position except that its axis is inclined to the right 60° to H.

(c) The same pyramid in the same position except that its axis is turned backward on the right 15° to V.

Problems in Development of Surfaces

Show H, V, and P, projections, auxiliary view, and development in the following problems. See pages 12, 13, 14.

Prob. 18. Truncated Square Prism. A square prism, with base parallel to H and the nearer side 30° to V, is cut by a plane 45° to H and perpendicular to V so that the prism is 3″ high on the left. The base is 1¾″ square.

Prob. 19. Truncated Triangular Prism. A triangular prism, with 2⅜″, 1½″, and 2⅛″ sides and the base parallel to H, has the broadest side at the rear and 15° to V. The prism is cut by a plane, 30° to H and perpendicular to V, so that it is 2⅝″ high on the extreme left.

Prob. 20. Truncated Hexagonal Prism. The hexagonal base is parallel to H and is 2″ across the flats. The prism is cut by a plane, 30° to H and perpendicular to V, leaving it 2¾″ high on the extreme left.

Prob. 21. Make the lay-out or 'stretch-out,' as it is termed in this case, of the mitred sheet metal moulding, the V projection of which is shown in Fig. 31. Scale, full size. Use only vertical projection and the bottom view.

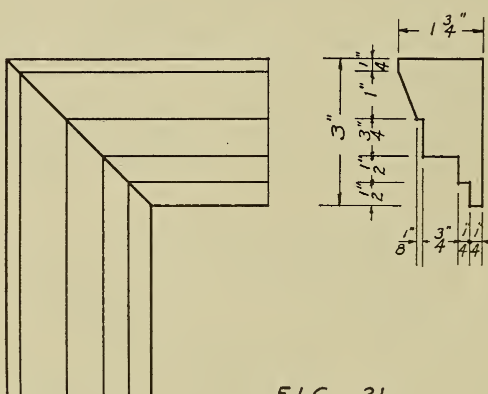

FIG. 31

Prob. 22. Truncated Square Pyramid. A square pyramid, with base parallel to H and sides of base 45° to V, is cut by a plane at 45° to H and perpendicular to V, ½″ above the base on the right-hand edge. The base is 1⅞″ square and the height of the original pyramid 3½″.

Prob. 23. Truncated Hexagonal Pyramid. An hexagonal pyramid, with the base parallel to H and the nearest side of the base parallel to V, is cut by a plane, 45° to H and perpendicular to V, ¾″ above the base on the right-hand edge. The base is 2¼″ across the corners; the original altitude 3½″.

Prob. 24. Truncated Oblique Square Pyramid. The base is parallel to H, its side 45° to V. the axis is parallel to V but inclined to the right so that the apex is $2\frac{1}{2}''$ to the right of the center of the base. The cutting plane, perpendicular to V, makes an angle of 45° with H and cuts the extreme right-hand edge $\frac{3}{4}''$ above the base. The sides of the base are $1\frac{7}{8}''$, the altitude of the original pyramid $3\frac{1}{2}''$. See page 15.

Prob. 25. Truncated Oblique Pentagonal Pyramid. The base is parallel to H with the right side of base perpendicular to V. The extreme left-hand edge is parallel to V and 45° with H. The base is inscribed in a $3''$ circle, the altitude is $3\frac{1}{2}''$, and the cutting plane is parallel to and $1\frac{1}{2}''$ above the base.

Prob. 26. Truncated Oblique Pentagonal Pyramid. The base is parallel to H with the left side of base perpendicular to V. The axis is inclined to the right so that the apex is $2\frac{1}{4}''$ to the right of the center of the base. The base is inscribed in a $2\frac{3}{4}''$ circle, the altitude is $3\frac{3}{4}''$, the cutting plane is perpendicular to V at any desired angle with H, so that the highest point is $1\frac{3}{4}''$ above the base on the extreme left-hand side.

Prob 27. A pyramid having its rectangular base, $1\frac{1}{2}''$ by $2\frac{1}{4}''$, parallel to H and being $3\frac{1}{2}''$ in altitude has the longer sides of the base parallel to V. The pyramid is cut by a plane, 30° with H and perpendicular to V, at a point $1''$ above the base on the right.

Prob. 28. Truncated Cylinder. A cylinder with base parallel to H is cut by a plane, perpendicular to V, so that the cylinder is $3''$ high on the extreme left-hand element and $\frac{1}{2}''$ high on the extreme right. The cylinder is $2\frac{1}{4}''$ in diameter. Show auxiliary view, profile view and development of curved surface. Study notes on the cylinder carefully. Page 12.

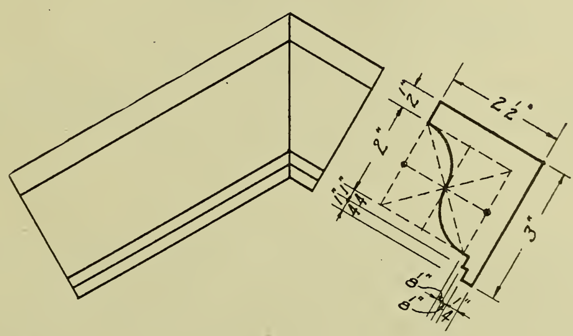

FIG. 32

Prob. 29. Make the 'stretch-out' of the sheet metal return, the bottom view of which is shown in Fig. 32. Scale, full size.

Prob. 30. Truncated Cone. A right circular cone, with base parallel to H and 2¾″ in diameter, is truncated by a plane, perpendicular to V and at 60° on the right with H, so that the extreme right-hand element is cut ½″ above the base. The original altitude of the cone was 3½″. Page 13.

Prob. 31. Truncated Oblique Cone. An oblique cone with a circular base, 2½″ in diameter, is 3½″ in altitude. Its apex is 2″ to the right of the center of the base and its axis is parallel to V, while its base is parallel to H. It is cut by a plane perpendicular to V and at an angle of from 45° to 60° with H. The cutting plane intersects the right-hand element ½″ above the base.

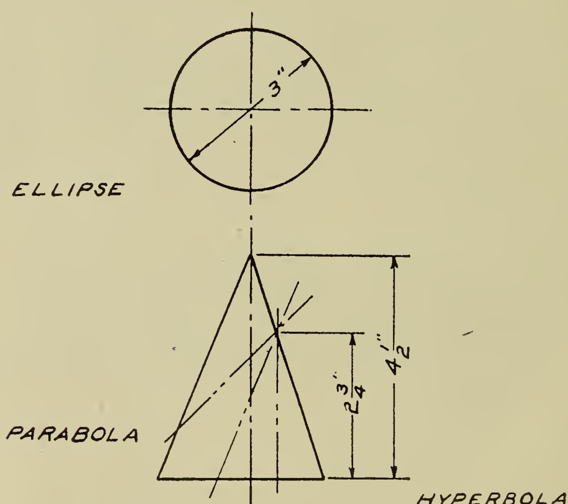

FIG. 33

Prob. 32. Conic Sections. Taking a right circular cone 3″ in diameter at the base, 3¾″ in altitude, cut it by planes through a point on the right 2¾″ above the base; the first parallel to the left-hand element, the second at about 45° to H, the third perpendicular to the base. All the planes are to be perpendicular to V. Find the revolved sections. Fig. 33.

Show all the necessary projections and the 'lay-outs,' or developments, complete. See pages 24 and 25.

Prob. 33. Pipe Tee. Diameter of main 2½″; of off-set 1¾″, axes in the same plane. Make the height of the larger cylinder 3″ with the axis vertical; and end of off-set 2″ from axis of main.

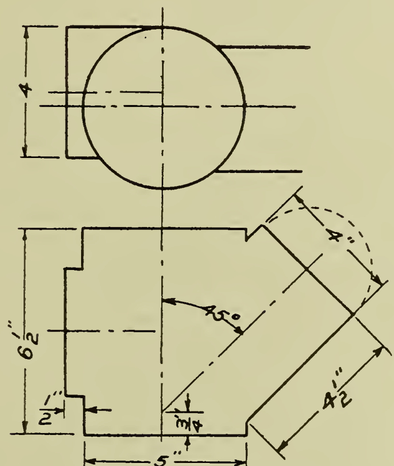

FIG. 34

Prob. 34. Sheet Metal Pipe Connection. Place the center of the H projection of the intersecting cylinders, indicated in Fig. 34, 7¼″ to the right of the left-hand border and 9¼″ above the lower border of a C-size sheet. Place the bottom of the V projection 4⅜″ above the lower border. At the right, show the profile view as well as the development of the oblique cylinder; at the left, show the profile view and the development of the horizontal cylinder; along the lower portion of the drawing show the development of the vertical cylinder. Scale, 6″=1′-0″.

NOTE:—An examination of the four-piece elbow, Fig. 35, will show that the truncated cylinders forming the end pieces are cut at a right angle with the axis on one end and at 15° to the base on the other. In the two intermediate pieces, the joint forms an angle of 15° with the center line of the piece. The angle of the elbow, 90°, is divided into six equal angles of 15°, formed by the joint between the pieces and the center line of the piece. For an elbow of any angle, there are as many angles formed as there are end pieces plus twice the number of intermediate pieces.

Example:—In a four-piece elbow there are,

2 end pieces	=2 angles
2 intermediate pieces	=4 angles
Sum of angles	=6

90°÷6=15° in each angle.

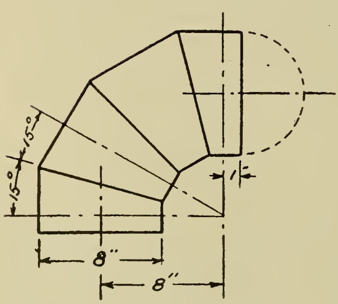

FIG. 35

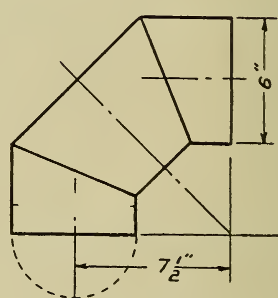

FIG. 36

Prob. 35. Four-piece elbow, Fig. 35. Scale, 4″=1′-0″.

Prob. 36. Three-piece elbow, Fig. 36. Scale, 6″=1′-0″.

Prob. 37. Five-piece elbow, Fig. 37. Scale, 4″=1′-0″.

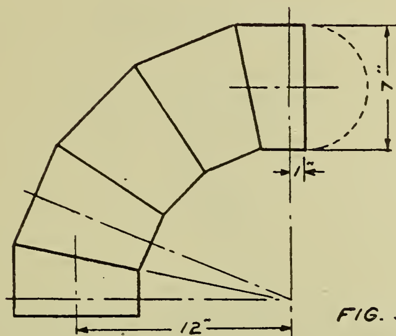

FIG. 37

Prob. 38. Grocer's Scoop, Fig. 38. Scale, 4″=1′-0″.

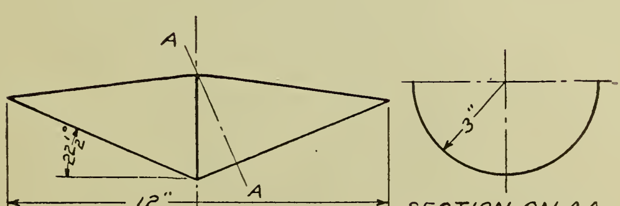

FIG. 38

Prob. 39. Funnel, Fig. 39. Scale, 6″=1′-0″.

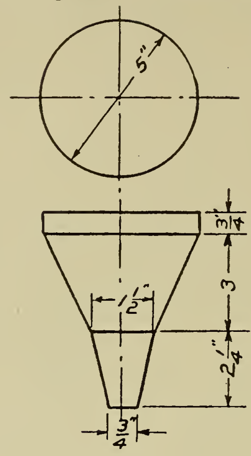

FIG. 39

Prob. 40. Ventilator, Fig. 40. Scale, 2″=1′-0″. Place the center of the H projection at 2¾″-8¾″, using a C-size sheet. The bottom of the cylindrical piece, in the V projection, 3¾″ above the lower border. In the lay-out of the bottom piece, make the seam down the longest side.

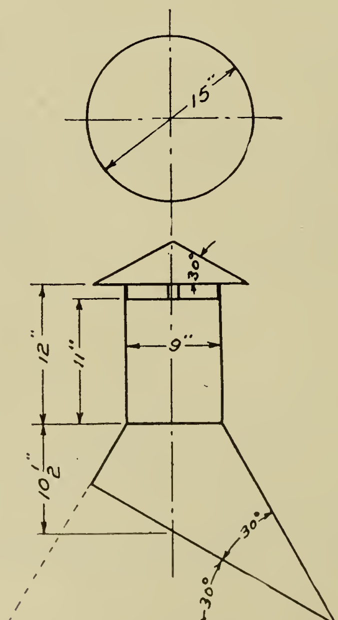

FIG. 40

Prob. 41. Transition Piece, Fig. 41. Scale, 3″=1′-0″. See second method of finding the true length of line, page 10.

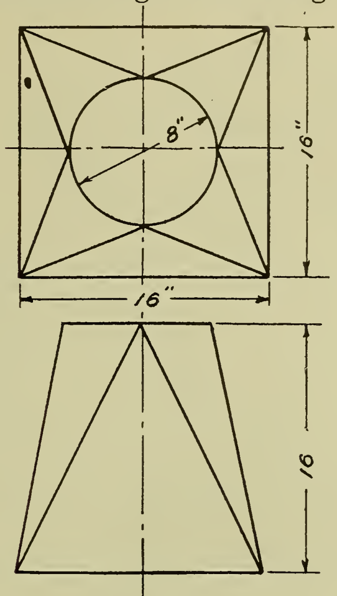

FIG. 41

Prob. 42. Transition Piece with off-set, Fig. 42. Scale, 3″=1′-0″.

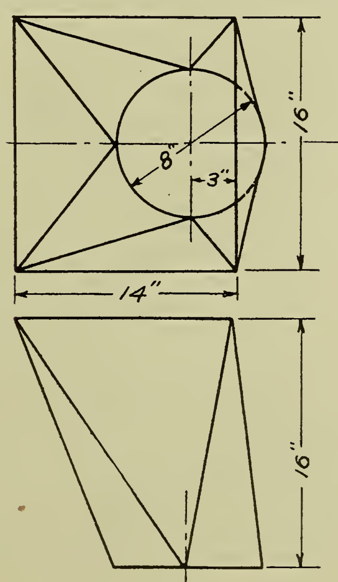

FIG. 42

Prob. 43. Connector, Fig. 43. Scale, full size.

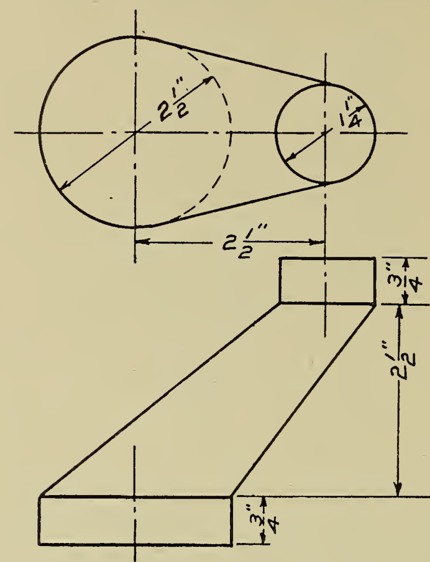

FIG. 43

Prob. 44. Connector, Fig. 44. Scale, full size.

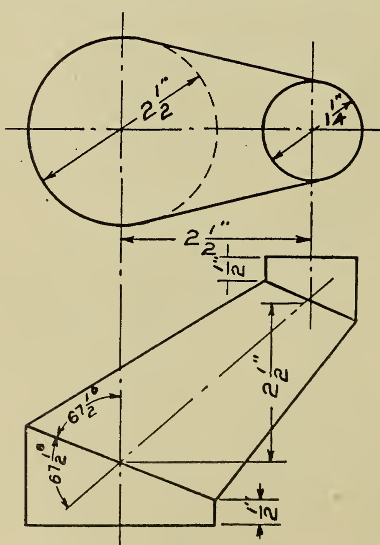

FIG. 44

Prob. 45. Reduction elbow, Fig. 45. Scale, 6″=1′-0″.

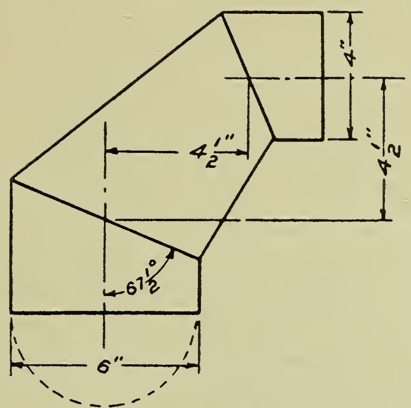

FIG. 45

Prob. 46. Oil-Bucket, Fig. 46. Scale, 6″=1′-0″.

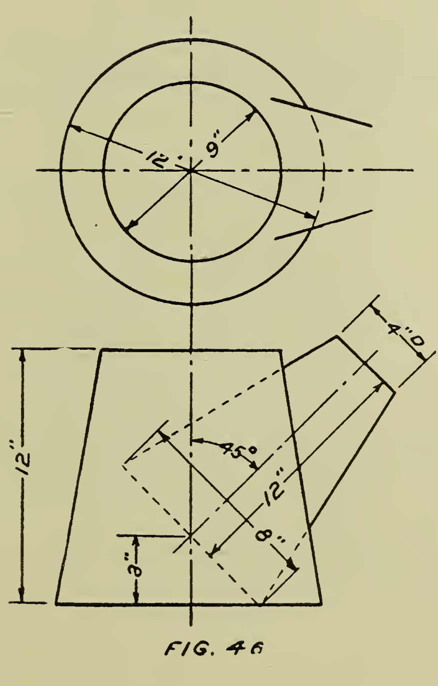

FIG. 46

Prob. 47. Intersecting Cone and Square Prism, Fig. 47. Scale, full size.

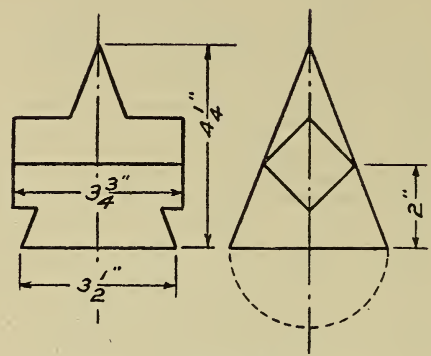

FIG. 47

Prob. 48. Intersecting Square Pyramid and Square Prism, Fig. 48. Scale, full size.

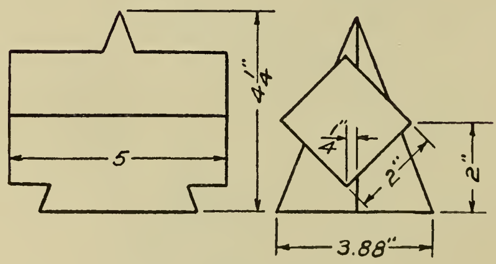

FIG. 48

Prob. 49. Make the 'lay-out' of the small tower shown in Fig. 49. Scale, 1″=1′-0″.

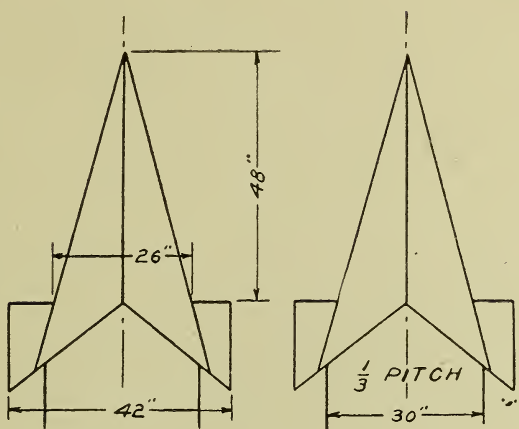

FIG. 49

Prob. 50. Find the true size of the dihedral angle formed by the sides of the lamp shade shown in Fig. 50. Scale, 3″=1′-0″.

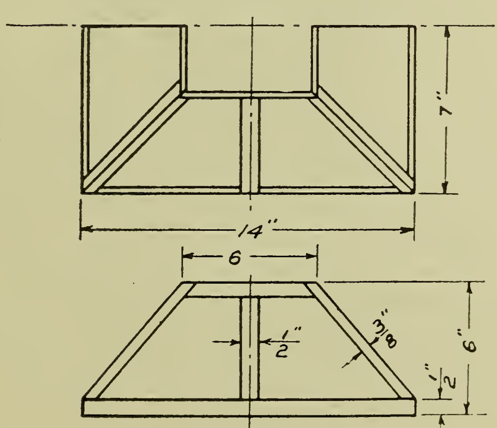

FIG. 50